CHAPTER 5

THE CHANCES OF LIFE AND DEATH

Summary: The average expectation of life is a point estimate obtained by summing the survival chances for all possible years of life to age 99 and beyond. The chance of inheritance is similarly calculated but with the chance of death in each year substituted for one of the survival ratios. The standard actuarial calculation for ascertaining present value proceeds by taking the value of the chance in each year of the relevant income or expenditure and then summing the resulting series of separate values. It is generally unsound to suggest that by consuming interest and capital a claimant may reproduce the income that has been lost.

[5.1] DEFINITIONS

The chances of life and death have been the subject of exhaustive analysis over the last 300 years. This chapter discusses the workings the risks of life and death. By actuarial standards the techniques discussed here are fairly elementary and well within the computational ability of a layman equipped with a modern microcomputer and a spreadsheet package.

[5.1.1] Period of survival: The single most prominent risk affecting the assessment of damages for personal injury and death is the contingency of life and death. It is usual to express this contingency as an average point estimate³ called the 'expectation of life'. This is expressed as a number of years but may be as little as few months⁴ or as much as 70 or 80 years. The expectation of life is popularly viewed as a prediction of when death will occur. This interpretation needs to be used with care because it can lead to seriously incorrect conclusions when dealing with marginal situations such as benefits after retirement and joint life expectancies.

Survival to advanced ages well in excess of 100 years has been recorded. For purposes of damages assessments it is generally adequate to assume age 99 to be the limit of life.⁵ The discussion below proceeds on this basis. The limit of life is the age beyond which the chance of survival is negligible. It is important to distinguish the limit of life from the 'expectation of life' in the statistical sense of an average.⁶ Popular usage of the expression 'expectation of life' sometimes has in mind the limit

¹See 84.

²Such as LOTUS 123.

³See 15.

⁴See, for instance, Gerke v Parity Insurance 1966 3 SA 484 (W).

⁵See Neill `Life contingencies' 7.

⁶See 80# and 81%

of life.

Death is an event which is foreseeable as a certainty. What is uncertain is when death will occur.

[5.1.2] Chance of survival: The chance that a person now aged 40 will survive to age 65 is calculated by taking from the life table the number of survivors at age 65 and dividing this by the number of survivors at age 40.7 The survival chance for a white female, for example, is 82,6%. It is assumed for purposes of the discussion that the life table used gives a fair indication of the true mortality risks.

[5.1.3] Chance of death: The chance that a person now aged 40 will die during the 65th year of age is calculated by differencing the survivors to ages 64 and 65 and then dividing by the number of survivors at age 40.8 The chance of death for a white female, for example, is then 1,5%.

[5.1.4] The expectation of life: This is best described as an index by which to compare one life table with another. It is calculated by adding up the separate chances of survival for each individual year between the present age, say 40, and age 99 or older, the limiting age of the life table which is being used. In this sense it is best described as the average duration of life because it is derived from life-table averages. Of particular importance in this regard is the concept of a series of chances of survival, one for each year between age 40 and age 99. These chances become less and less with advancing age and reduce to nil at the end of the life table. The importance of these chances is that they permit a calculation of a separate value of the chance of survival to each year between age 40 and age 99. The expectation of life falls between age 40 and age 99. The standard actuarial calculation is not terminated at the expiry of the expectation of life but at age 99, the limit of life.

The expectation of life is not generally used by actuaries.¹⁰ The standard actuarial calculation proceeds on the basis of the year-by-year application of the value of the chance of survival to each relevant year.¹¹ Much of actuarial science is devoted to techniques for arithmetically manipulating these complex contingencies.

[5.1.5] Reduced expectation of life: The evidence may indicate that a claimant has a

⁷Using SALT79/81 white females we have 79077/95760=0,826. This may be expressed as a percentage by multiplication by 100 giving 82,6%.

⁸SALT79/81 white females (80542-79077)/95760=0,0153 which is 1,53%. The chance of death between age 40 and 65 is 1 minus the chance of survival to age 65, ie 1-0,826=0,174, ie 17,4%.

⁹See example in table 5 at 88. Strictly speaking this gives a particular type of expectation called the 'curtate' expectation (see Neill 'Life Contingencies' 201-2). The reader need not concern himself with the fine distinctions between the different types of expectations. The main point to grasp is that they are obtained by a summation of separate chances of survival, usually on a yearly basis.

¹⁰Prevett 1972 *MLR* 140 147. The standard text for trainee actuaries mentions it only briefly under a chapter headed 'Population theory' (see Neill 'Life Contingencies' 201-2).

¹¹Newdigate & Honey `MVA Handbook' 167; Carstens v Southern Insurance 1985 3 SA 1010 (C) 1024G-H. See 88 below.

reduced expectation of life.¹² **This means that the chance of early death has been increased** and that the life table must be recalculated so that the survival chances for each separate year add up to the reduced life expectancy. The adjustment to the chances of death will often be expressed in the form 'plus 100% extra mortality',¹³ for example, that is to say the risk of death in any one year is doubled. Hrubec & Ryder report 40% more deaths for persons with a limb amputation.¹⁴ For serious brain injuries Roberts reports studies which indicate a reduction in life expectancy of about 4 years.¹⁵ For paraplegics, quadriplegics and epileptics the risk of death due to their condition does not increase with advancing age although the underlying risk of death does increase, as for any normal member of the population. For such persons it is preferable to adjust the life table by a constant addition to the risk of death such as 1% of survivors at the beginning of each year.¹⁶

Consider a white female aged 40 whose life expectancy has been reduced by 10 years. Had she not been injured her expected age at death would have been 76,75 years. Now that she is injured her expected age at death has been reduced to 66,75 years. Some analysts might conclude that her reduction in life expectancy does not affect the value of her earnings up to age 65. This would be an invalid conclusion because, as has been stated above, reduced life expectancy implies that the risk of early death has been increased. The reduction of 10 years to life expectancy would in fact reduce the value of the claim for loss of earnings by 8,5%. ¹⁸

[5.1.6] Anecdotes and averages: Each individual has a personal perception of survival, the utility of the duration of life. The biblical three score years and ten probably colours this perception to a substantial degree. For many younger persons the difference between a prospect of death at age 70 or age 170 is of scant significance, so remote is it in time. For some persons the expression `expectation of life' implies

¹²See, for instance, Carstens v Southern Insurance 1985 3 SA 1010 (C) 1024-27.

¹³An actuary would probably prefer to see this expressed algebraically in the form +k%.

¹⁴Hrubec & Ryder 33 (1980) *J Chron Dis* 239-50. `The lack of physical exercise makes him a candidate for coronary artery disease and obesity' *August v Guardian National Insurance* 1990 4 C&B E2-13 (C) 14.

¹⁵Roberts 'Severe accidental head injury' 1979 148-51. Earlier in the same chapter Roberts attempts to analyze extra mortality by separating normal deaths from death directly caused by brain injury. This approach is statistically unsound in that it presumes that medical practitioners are capable, many years after the event, of accurately identifying the cause of death. Brain injury has a variety of subtle life threatening effects, one of these being to reduce the victim's ability to identify his own illnesses and to manage them. The scientifically correct approach to measuring mortality is to take a population of brain injured persons and compare the number of deaths from this population with the number of deaths in the normal population.

¹⁶Thus if the normal risk of death in a 12-month period is 4% then the adjusted risk is 5% (4%+1%). If the normal risk of death in a subsequent 12-month period is 6,5% then the adjusted risk is 7,5% (6,5%+1%), and so on. For more detail see Fisher & Young `Life assurance' 134-9.

¹⁷SALT79/81 females (40+36,75).

¹⁸Using 2,5% per year net capitalization rate (see 130) and an extra mortality of 173%. *Lockhat's Estate v North British & Mercantile Insurance* 1959 3 SA 295 (A) ruled that when assessing loss of earning capacity the calculations for both the injured and uninjured conditions should be based on the reduced life expectancy (see 225 below).

the limits of life, age 90 or 100 and beyond. Such longevity is, however, only a remote possibility. For compensation purposes one needs to objectivize the expectation of life¹⁹ and balance the prospect of possible longevity against possible early death. This implies the concept of an average expectation of life.

Medical experts are often consulted as regards the effect of injuries on the expectation of life. Not all such experts are astute to use the average expectation and one quite frequently finds opinions as to expectation of life couched in anecdotal terms: 'I know of a paraplegic who has lived the normal span. A normal lifespan is possible therefore all paraplegics have a normal life expectancy'. In practice the average expectation of life for paraplegics is below normal.²¹

Another form of medical opinion that gives rise to some degree of confusion arises when there is, for example, a 20% risk of epilepsy. The opinion will often be expressed that provided epilepsy does not occur the expectation of life is normal. The fact of the matter is that if there is a risk of epilepsy then there is an associated increased risk of early death, ²² and thus an immediate reduction to life expectancy. The risk exists even if epilepsy as a certainty does not.

Not all evidence leads to reduced life expectancies. Evidence of a family history of longevity may justify a longer-than-normal life expectancy.²³

Medical opinion concerning life expectancy, if it is to be accurate, should specify the life table considered normal and the effect in relation thereto of the victim's condition. Most medical experts have only limited access to life tables and the interpretation thereof and their opinions should ideally be formulated in consultation with an actuary, as is done by medical underwriters at a life office. The focus of medical inquiry in relation to damages claims is usually on whether the life expectancy of the victim has been reduced. It is often relevant, however, to consider the effect on life expectancy of pre-existing conditions such angina, a history of heart attacks, multiple sclerosis, diabetes, tuberculosis, etc. The question of AIDS has to date not received much attention from the courts.

[5.2] HISTORICAL BACKGROUND

[5.2.1] Ulpian's table: The life table recorded by Ulpian²⁴ is of considerable historical

¹⁹See 22.

²⁰This approach equates life expectancy with the limit of life.

²¹Walsh & Yeo 1985 FESPIC 142 report for Australia under optimal care conditions a 5% reduction for paraplegics and a 15% reduction for quadriplegics. Geisler (1983) 21 Paraplegia 364 reports very much heavier rates of mortality. The two papers both cover much the same period of time. The observed differences are thus not explained by improvements in medical science. Walsh & Yeo have probably observed a very much more affluent and better educated sector of society.

²²Brackenridge `Life Risks' 2ed 604-8 reports extra mortality for epileptics ranging from nil to +200%. Laidlaw & Richens `Epilepsy' 28 report for the USA that `the rates are significantly higher for non-whites. This applies particularly to males for whom the rates are three times as high as those for whites'.

²³Nochomowitz v Santam Insurance 1972 1 SA 718 (T) 721-2.

²⁴D35.2.68.

importance. For over 1000 years it was the primary reference for length of life for jurists following the Roman-law tradition.²⁵ Kopf²⁶ suggests that it may reflect the mortality experience of the mutual aid societies which provided pensions to Roman legionaries after their retirement at age 46.

Age	Multiplier
0-20	30
21-25	28
26-30	25
30-35	22
36-40	20
40-50	60 minus age minus 1
50-55	9
55-60	7
61 and over	5

TABLE 3 - ULPIAN'S LIFE TABLE

An abridged version provides for the expectation to be taken as 30 years up to age 30 and thereafter 60 minus age with 5 years being used for anyone over age 55. This rule-of-thumb was popular with jurists.²⁷

There is reason to believe that Ulpian's table records not life expectancies but annuity factors which include a discount for interest: The original purpose of Ulpian's table was to capitalize usufructs over property for estate duty purposes. The value of a perpetuity²⁹ is based on a multiplier of 30 years. This implies a net capitalization rate of 3,3% per year. For young persons Ulpian's table limits the period to 30 years, falling far short of the biblical `three score years and ten'.

[5.2.2] Modern life tables: The intuitive perception of a life table is that of a table of life expectancies. Life expectancies are difficult to measure directly and do not lend themselves to sophisticated mathematical treatment. The introduction in 1662³⁰ of a life table based on survivors at selected ages anticipated the flexible modern life

²⁵Davel `Broodwinner' 7-17. The word `*lijfrente*' used by Grotius `*Inleiding*' 3.33.2 is a reference to Ulpian's table.

²⁶Kopf (1927) 13 PCAS 225 232 233.

²⁷See, for instance, Matthaeus `De Criminibus' 47.4.5 48.7.11; Azo `Institutiones' 4.4.11.

²⁸D35.2.68 is concerned with `The law of 5% tax of estates'.

²⁹Regular yearly, or more frequent, payments which never cease.

³⁰By John Graunt. His table based on christenings and burials in the City of London was too imprecise to be of any real value (Dublin & Lotka `Length of Life' 40-2; Benjamin & Haycocks `Analysis of Mortality' 385-9).

table. The SALT79/81 table³¹ shows for a notional 100000 born the number surviving to each age up to age 90.³² From this table of survivors may be calculated related factors: the chance of survival, the chance of death, the expectation of life. Most life tables are constructed from direct observations of death rates over fairly short periods of time and the associated lives exposed to the risk of death, what one might call a 'snapshot basis'.³³

[5.3] RESTITUTIO IN INTEGRUM

Consideration of the expectation of life as a number of years free of the risk of death can lead one into the error of thinking that something approximating perfect restitution is possible. I have already pointed to the construction of the expectation of life from the accumulated sum of intervening chances of survival.³⁴ I will now examine some further features of the chances of death and survival.

Attained Age	Expected Age at Death
40	76,75
60	77,54
70	79,33
75	80,96
80	83,26

TABLE 4 - EXPECTED AGE AT DEATH

SALT79/81 white female mortality

[5.3.1] `End-of-the-rainbow' phenomenon: The most notable feature of the expectation of life is that as a person gets older the expected age at death advances into the future.³⁵ This point is illustrated in table 4.

The expected age at death is thus like the pot of gold at the end of a rainbow, fleetingly unattainable. This phenomenon has the important consequence that if a plaintiff has been compensated at age 40 on the basis of consuming interest and capital to replace the required income³⁶ and acts precisely in accordance with this

³⁵Boberg 1963 SALJ 538 545n29; Davel `Broodwinner' 507n536 record this phenomenon.

³¹Department of Statistics (whites coloureds asiatics); the 1984-86 tables are those most commonly in use for compensation purposes in 1993 (see *Quantum Yearbook* 1993 72-83). These are differ little from the older tables.

³²For reasons of convenience. These tables extrapolated to age 99 appear in 1986 *De Rebus* 551 552-4. Survival beyond age 99 is rare but not unknown.

³³Cox `Demography' 198-200. Life offices are able to use more sophisticated methods due to their extensive policyholder data (see Benjamin & Haycocks `Analysis of Mortality' 35-51

³⁴See table 5 at 88.

³⁶See Gillbanks v Sigournay 1959 2 SA 11 (N) 15A; SA Eagle Insurance v Hartley 1990 4 SA 833 (A) 838-9.

directive then the capital will be exhausted by age 77.37 By that time the expected age at death will have increased to over 81 years.

From a statistician's point of view the advancing of the expected age at death to older ages would be described as a Bayesian revision of the expected age at death based on the new information that the plaintiff had actually survived to an older age.³⁸ Jurists would describe this procedure as 'taking account of events supervening between date of death and date of trial'.³⁹

[5.3.2] Expectation of working life: The vast majority of compensation matters are concerned with the working lifetime of the plaintiff. It is usual to assume retirement at age 65 although ages ranging from 45 to 80 and beyond are encountered in practice. 40 The standard actuarial calculation includes a substantial deduction for the risk of early death. It follows that the lost income cannot be reproduced by consuming interest and capital over the expectation of working life as calculated by an actuary. The same conclusion follows from the end-of-a-rainbow phenomenon.⁴ Actuaries testifying in court have experienced singular difficulty with explaining the process of discounting for risk for a single individual.⁴² Actuarial literature records the following hypothetical exchange in court:⁴³

^{&#}x27;Judge: "Just a moment, Mr Actuary, I don't quite follow that line of argument. You say that \$300 will replace a \$1 per week over the lifetime⁴⁴ of the plaintiff or until he attains

age 65, whichever is earlier."

'Actuary: "Yes, Your Honour."

'Judge: "But what happens to the \$300 if the plaintiff dies in 6 months time?"

'Actuary: "The greater part of the \$300 will be remaining, Your Honour."

'Judge: "But you said that \$300 would provide the income just over the plaintiff's lifetime." Here follows a long circular explanation by the actuary about averages etc which confuses the judge even more and raises the actuary's pulse.

^{&#}x27;Judge: "Well, just let us leave that argument aside for one moment. Let us examine the

³⁷Provided earlier death has not intervened.

³⁸Savage 'Bayesian Econometrics' 44 60 65.

³⁹Wigham v British Traders Insurance 1963 3 SA 151 (W) 155-6: Plaintiff aged 81 with an expected age at death of 87 survived pre-trial period of 3 years and then had an expected age at death of 89 \(\)... the Court is entitled in the case of prospective damages to inform itself of subsequent facts which are known at the date of the trial and which if taken into account would enable the Court to determine with a greater degree of certainty or accuracy the actual loss of a plaintiff.

⁴⁰Davel `Skadevergoeding' 108 comments on the general acceptance by the courts of age 65 as standard retirement age and the absence of evidence to indicate other retirement ages. The court records reflect only a very small proportion of claims. My own experience is that a wide variety of retirement ages are used and that allowance will generally be made for a post-retirement pension, if not for post-retirement employment.

⁴¹See paragraph 5.3.1.

⁴²Younger actuaries are these days receiving training in utility theory. One may thus expect to see utilitarian reasoning advanced in years to come.

⁴³Crocker 1980 *TIAA* 517 586-7.

⁴⁴The word `lifetime' is here clearly intended to mean `working lifetime', ie expectation of working life as distinct from the full expectation of life. This looseness of terminology is common in both South Africa and, it seems, Australia.

situation in say 20 years' time⁴⁵, Mr Actuary, which you state in your certificate is the life expectancy of the plaintiff. Will the income be provided by the \$300 for this period?" `Actuary will probably say - to simplify the argument: "Yes, Your Honour". `Judge: "And what part of the \$300 will then be remaining?"

'Actuary: "Very little, Your Honour."
'Judge: "Well, from where is the plaintiff going to obtain his \$1 per week until age 65, which is then still 5 years off?" At this stage the actuary gives up'.

The 'expectation of working life' discussed above is calculated in the same way as the full expectation of life save that chances of survival after age 65 are ignored. The expectation of working life terminates several years before normal retirement age because it includes allowance for early death. Thus for a coloured male aged 40 the period to age 65 is 25 years but the expectation of working life is 19,88 years, that is to say it expires just before the 60th birthday.

[5.3.3] Contingency of early death: The above quotation discusses a possible working lifetime of 25 years with an expected working lifetime of just short of 20 years. This reflects a deduction for the contingency of early death of 20%. This deduction is made by the actuary as part of his calculations. The court will usually make a further deduction for general contingencies of about 10%, giving a total deduction for all contingencies of about 30%.46 We can observe here an application of valuation of a chance. We have earnings as a certainty calculated over 25 years less 20% for the chance of early death⁴⁷ less a further 10% for other contingencies.

[5.3.4] To put in the position he would have been in: The 'gross multiplier method'48 envisages a yearly payment discounted at interest over a period, the expectation of life⁴⁹ or the expectation of working life.⁵⁰ By this means the courts often presume to effect *restitutio in integrum*, an exact replacement of the lost income.⁵¹ The phenomenon of increasing expected age at death, the end-of-a-rainbow anomaly, demonstrates that the gross multiplier approach does not achieve perfect restitution. A similar problem is apparent when one uses a working lifetime discounted for the

⁴⁵Crocker's paper uses the period 23 years. I have replaced this with 20 years to bring the flow of the argument into line with the calculation example given below this quotation.

⁴⁶Strictly speaking 28% (0,28=[1-0,8x0,9]).

⁴⁷A number of writers have pointed to the failure by the courts to appreciate that a calculation by an actuary includes allowance for the contingency of early death: Boberg 1964 SALJ 194 204n54; Street 'Damages' 120; Luntz 'Damages' 2ed 280; see, for example, Ncubu v NEG Insurance 1988 2 SA 190 (N) 193H 198A. Not all courts miss the point, see Bester v Silva Fishing Corp 1952 1 SA 589 (C) 600B 'In the determination of the expectation of life due regard has been had to the probability of earlier demise' ('probability' here used in the sense of a chance less than 50%).

⁴⁸Boberg 1964 *SALJ* 194 204-5; Davel `Broodwinner' 511; Koch `Damages' 47.

⁴⁹See, for instance, Bester v Silva Fishing Corp 1952 1 SA 589 (C) 600B.

⁵⁰Nhlumayo v General Accident Insurance 1986 3 SA 859 (D) 861I-J.

⁵¹Nhlumayo v General Accident Insurance 1986 3 SA 859 (D) 861I-J `He was quite emphatic that his method was the way of putting the plaintiff in exactly the same position as he would have been if there had been no accident' (emphasis supplied). In this matter the court was concerned with the expectation of working life (at 8611) 'The sum which invested will produce an annuity which would theoretically expire in the course of his working life'. See too Crocker 1980 TIAA 517 586-7.

risk of early death. The actuarial year-by-year method⁵² demonstrates that the end-of-a-rainbow problem is but one manifestation of the more fundamental problem with the value of a chance. The result is always the same - the compensation money will either be too much or too little but seldom, if ever, just right. Precisely how we can explain and justify a measure of damages based on an average, the expectation of life at the time of making the award, requires a closer look at utility theory.⁵³ Utility theory, as I have already noted, indicates that restitution is achieved in terms of a 'price in a manner of speaking' representing the present utility of the future income, that is to say the lump-sum payment. The claimant's present lump-sum utility is restored. Restitution is not achieved in terms of the future income represented by that lump sum.

[5.3.5] The risk of living too long: There is a substantial risk, of the order of 50%, that a plaintiff will survive beyond the age indicated by his expectation of life. The prudent plaintiff⁵⁴ should have regard for the likelihood⁵⁵ that he will outlive his original actuarially determined expectation of life. Such a plaintiff should have the sense to ignore suggestions that he consume all capital by the expiry of his original expectation of life.⁵⁶ A safer investment policy would be directed towards preservation of a substantial proportion of capital throughout life in order to provide for the contingency of longevity, like any other person.

⁵²Newdigate & Honey `MVA Handbook' 167; Carstens v Southern Insurance 1985 3 SA 1010 (C) 1024G-H.

⁵³See 5 et seq.

⁵⁴The question of risk averse and risk seeking personalities has already been discussed (see paragraph 2.4.1).

⁵⁵ Likelihood' is used here in the sense of `certa spes'.

⁵⁶ie that prevailing at the time that the award is made.

Gross multiplier method Year-by-year method Survival Interest Interest Discount Value Discount Value Age Age R10000x R10000x 16%py 16%py A R AxB CxD 1.000 0.928 9285 0.995 0.928 9237 40 40 0.984 7878 41 1.000 0.800 8004 0.800 41 42 0.973 1.000 0.690 6900 0.6906713 42 43 1.000 0.595 5948 0.961 0.595 43 5717 44 1.000 0.513 5128 0.949 0.513 4864 44 45 1.000 0.442 4421 0.935 0.442 4135 45 46 1.000 0.381 3811 0.922 0.381 46 3512 0.907 2979 47 47 0.329 3285 0.329 1.000 48 1.000 0.283 2832 0.891 0.283 2524 48 49 2441 49 1.000 0.244 0.875 0.244 2135 50 0.210 2105 0.857 0.210 1803 50 1.000 51 51 1.000 0.181 1814 0.838 0.181 1520 52 1.000 0.156 1564 0.818 0.156 1279 52 53 1.000 0.135 1348 0.796 0.135 1074 53 54 54 0.774 1.000 0.116 1162 0.116 900 55 55 1.000 0.100 1002 0.752 0.100 753 $0.72\overline{8}$ 56 629 56 1.000 0.086 864 0.086 57 1.000 0.074 745 0.704 0.074 525 57 58 58 0.680 1.000 0.064 642 0.064 437 59 489 0.655 59 0.884 0.055 0.055 363 60 0.000 0.000 0 0.630 0.048 301 60 61 0.000 0.000 0 0.605 0.041 249 61 62 0.000 0.000 0 0.578 0.035 205 62 63 0.000 0.000 0 0.552 0.031 169 63 64 0.000 0.000 0 0.525 0.026 138 64 Σ 19,884 R63790 R60039 Σ 19,884

TABLE 5 - GROSS MULTIPLIER & YEAR-BY-YEAR METHODS

Note that due to rounding problems the total of 19,884 does not exactly match the total of the rounded figures shown in the table.

[5.4] THE ACTUARIAL YEAR-BY-YEAR METHOD

[5.4.1] Sliding-scale survival chances: We have observed that the modern life table is based on the notion of survivorship, the number alive at a selected age from an original hypothetical cohort of 100000 who were all born at the same moment. From this table may be calculated the proportion of those now alive who are expected to survive to a specified later age. Thus the SALT79/81 table for coloured males shows a figure of 77361 for survivors to age 40 and 39557 for survivors to age 65. The proportion expected to survive to age 65 is thus 51%, ie 49% of the group are expected to die before attaining age 65. It would be wrong however to assume from this statistic that a deduction of 49% should be made for the contingency of death prior to age 65. The expected working life from age 40 to age 65 based on the individual survival chances in each year is 19,88 years. This indicates a deduction

of about 20% for mortality prior to age 65,⁵⁷ in lieu of the 48,9% indicated by looking at survivors to age 65 only. The reason for the difference is that we are concerned with the average risk of death over the period.

[5.4.2] Yearly slices: The figures of the previous paragraph emphasise the need to cut the calculation into 'slices' which take account of the time at which the financial benefit is to be received. As a general rule earnings are received weekly or monthly. Life tables tend to work with yearly 'stops'. The practical 'slice' is generally a yearly one. The value of the chance that earnings of R10000 will be received at age 65 is 51% of R10000, ie R5100. A similar calculation may be done for the value of earnings at ages 64, 63, 62, etc down to age 40. These individual values of a chance are summed to give the total value for R10000 per year over the entire period.

[5.4.3] An example: The calculation is illustrated in table 5⁶¹ using a yearly payment of R10000⁶² which remains fixed in nominal terms, that is to say does not increase with inflation, a discount rate of interest of 16% per year, and SALT79/81 coloured male mortality. The table also shows a comparable calculation using the 'gross multiplier method'. The reader should note under column C how the expectation of working life of 19,884 years is obtained by adding up the individual chances of survival to each intervening year. For the gross multiplier method I have shown under column A the survival chances which are implicit to that cruder method⁶⁶. The total rand value using the gross multiplier method is R63791 whereas that using the

⁵⁷19,88/25=0,795, ie 79,5%. 100%-79,5%=20,5% which is 20% in round figures (SALT79/81 coloured male).

⁵⁸The mathematics of life contingencies discusses the problem in terms of infinitesimals and the `force of mortality' (Neill `Life Contingencies' 14-19).

⁵⁹De Witt in Holland used half-yearly stops in his presentation to the States General made in 1671 (Bouwstoffen `Levensverzekeringen en Lijfrenten' 5-6). This seems to be the earliest recorded use of the year-by-year technique. De Witt's work was lost during the subsequent political upheaval. The year-by-year method first obtained general public recognition from the writings of De Moivre and Simpson during the years 1740-44.

⁶⁰For sake of clarity of argument I have ignored the discount for interest. This is equivalent to an assumption that the discount rate of interest equals the expected rate of inflation. The former cancels out the latter. If no discount is made for interest then the gross multiplier and year-by-year methods yield identical results. The higher the net capitalization rate the greater the difference between the two methods.

⁶¹At 88.

⁶²For reasons of convenience I have here assumed that the payment is made in the middle of each year. Payments made monthly or weekly may for calculation purposes be conveniently replaced with a single payment at mid-year for the same total amount.

⁶³Such payments are not uncommon in compensation matters: Housing subsidies often take this form. Retirement annuities (pensions) in payment are often of a fixed monthly or yearly amount. It is common to find maintenance payments which do not increase.

⁶⁴The chance of survival to the middle of a year is approximated by the average of the chances of survival to the beginning and end of that year.

⁶⁵Boberg 1964 SALJ 194 204-5; Davel 'Broodwinner' 511; Koch 'Damages' 47.

⁶⁶100% certainty of survival prior to expiry of the expectation of working life, 0% chance thereafter.

year-by-year method is R60038. I have deliberately used a high net capitalization rate (16% per year) in order to emphasise the different results produced by these two methods.⁶⁷ When the calculation is done over the whole of life the differences become very much more pronounced.⁶⁸ The lower figure of R60038 would be considered by actuaries worldwide to be the preferable value.⁶⁹ It gives proper weight to the timing and contingencies attaching to the relevant payments.

[5.4.4] Restitution of income: What is evident from table 5⁷⁰ is that the individual yearly values, the yearly 'slices' reflecting the value of the chance of the income, have each been reduced for the contingency of early death. An important observation is that each 'slice' if invested diligently to produce investment returns equal to the discount rate of interest will be inadequate when the time comes to fully replace the income lost. This is so because of the deduction for the risks of death and general contingencies. The overall inadequacy is evident if one considers survival by the claimant to age 65.⁷¹ For children who have lost support the deductions for the mortality of the parent during dependency are often of minor consequence (perhaps 1% or 2%).⁷² It can then be correct for practical purposes to speak of consuming interest and capital to replace the lost support. It is otherwise with the example in table 5.⁷³ Restitution, as has already been noted, is achieved under lump-sum compensation in terms of present utility but not in terms of the future income represented by that present value.

[5.4.5] Contingency funds: The notion of consuming interest and capital has in mind the familiar home loan repaid by regular instalments, otherwise known as a 'sinking fund'. In its ideal form there is no risk or other uncertainty attaching to either the payments or the capital. When the payments are subject to the contingencies of human life and the accidents of employment and inflation there is no neat relationship between the payments and the present value of those payments. A fund established to cover uncertain future payments is appropriately termed a 'contingency fund'. The reserves held by life offices and pension funds to secure their future contingent liabilities are calculated by actuaries and are correctly described as 'contingency funds' although actuaries do not use such terminology. Actuaries who

⁶⁷As the effective discount rate of interest reduces the difference becomes smaller. For a nil discount rate of interest there is no difference at all. Substantial differences can arise with uneven cash flows, eg where allowance is made for major promotions.

⁶⁸See comparative tables in Koch `Damages' 304 and worked examples 257-91.

⁶⁹Kemp `Damages' 3ed 103; Newdigate & Honey `MVA Handbook' 167; Milburn-Pyle & Van der Linde 1974 *TASSA* 292 298; Street `Damages' 118; Luntz `Damages' 2ed 281; Davel `Skadevergoeding' 98-9; Crocker 1980 *TIAA* 517 576; *Snyders v Groenewald* 1966 3 SA 785 (C) 789sup; *Carstens v Southern Insurance* 1985 3 SA 1010 (C) 1024G-H; Koch `Damages' 46-7; Koch 1982/83 *TASSA* 78 87 (note comments by De Bruijn at 107-9 and De Bruijn's use in the *Carstens* matter of a year-by-year approach; Davel op cit 98n602).

⁷⁰At 88.

⁷¹See paragraph 5.3.2.

⁷²This was the example used in *General Accident Insurance v Summers* 1987 3 SA 577 (A) 613-14 to validate a compensation model based on consuming interest and capital.

⁷³At 88

calculate these reserve funds are described as 'valuators'⁷⁴ but this is a most misleading expression because the amounts calculated are not values for the exchange of goods in a market. Strictly speaking the actuary is not a valuator but a financial manager. For a life office or pension fund many lives are involved and statistical averaging gives rise to substantially predictable cash flows. A contingency fund for a single individual does not have the benefit of statistical averaging and requires very different reserving techniques. I will discuss below the concept of an overfunded reversionary trust,⁷⁵ a contingency fund appropriate to providing a future contingent cash flow for a single individual.

[5.4.6] Present utility: In relation to a single individual claimant the actuarial year-by-year method with its deductions for risk based on averages is, as I will discuss further below, a tool of utility analysis, the determination of a present financial equivalent for the utility of the life plan which has been lost. It is a price for which the plaintiff foregoes the right to claim further compensation.

[5.4.7] Retirement benefits: The example in table 5⁷⁶ assumes that no further income would have been received after age 65. This is generally a false assumption. Many employers today provide pension funds. Those without such benefits must rely on their savings or a state pension or continue working. Statistical conclusions based on the expectation of life can be misleading: Thus for a coloured male aged 40 the expected age at death is 65,3.⁷⁷ It would be incorrect to reason that such a man has the expectation of only 0,3 years of retirement.⁷⁸ The actuarial year-by-year method reveals that for a coloured male aged 40 now the correct present value of a pension of R5000 per year from age 65 onwards is R12210.⁷⁹ The present value of earnings of R10000 per year prior to age 65 is R153014. The total for earnings and pension is then R165224, the pension making a substantial 7,4% of this figure.

[5.4.8] Joint survivorship: A claim for damages by a widow for loss of support requires consideration of the joint survivorship of the breadwinner and the widow. The mortality of healthy children has little financial significance and it is usual to

⁷⁴s10 of the Insurance Act 27 of 1943.

⁷⁵See 108.

⁷⁶At 88.

⁷⁷SALT79/81 coloured males.

⁷⁸As in *Quntana v Union & SWA Insurance* 1976 2 C&B 680 (E) 682 2nd paragraph. In *Reid v SAR&H* 1965 2 SA 181 (D) 190-1 the court incorrectly chose to ignore increased mortality because the expectation of life after reduction still exceeded age 65. When there is increased mortality the chance of death prior to retirement increases and a larger deduction needs to be made for pre-retirement mortality. Crocker 1980 *TIAA* 517 572 observes that 'Knowing the misuse to which life expectancies are prone to be put by lawyers I would be loath to quote a value for the expectation of life without first soliciting information on the use which is to be made of it'.

⁷⁹Using a net capitalization rate of 2,5% per year and the SALT79/81 table for coloureds. Pensions are normally capitalized at a somewhat higher rate of 5,5% per year for the period after retirement.

⁸⁰Clair v PE Harbour Board (1886) 5 EDC 311 318; Hulley v Cox 1923 AD 234 245. See Davel 'Broodwinner' 522n591 for numerous other instances.

ignore it. The chance that a husband and wife⁸¹ both aged 40 today will both survive to age 65 is the product of the survival chances for each spouse separately. For a white male aged 40 the chance of survival to age 65 is 0,69, ie 69%. For a white female of the same age the chance is 0,83, ie 83%. The chance that they will both survive to age 65 is 0,69x0,83=0,57, ie 57%. The value of dependency during the years up to age 65 would be based on a year-by-year calculation, with the joint survivorship chances substituted in column C of table 5⁸⁴ for each possible age 64, 63, 62, etc back to age 40. By adding up these survivorship ratios one obtains the joint expectation of working life. The joint survivorship ratios may be calculated for all possible ages up to when the oldest spouse attains age 99, the limit of life. The sum of these ratios provides the joint expectation of life. The joint expectation of life is always less than the lesser of the individual expectations of life. This is so because the individual survival ratios for one spouse are each reduced by the corresponding survival ratios for the other spouse.

[5.5] CHANCE OF INHERITANCE⁸⁹

[5.5.1] Gross multiplier method: ⁹⁰ The value of a spes of inheritance depends on a number of factors including the survivorship of the wife and the chances of the husband's death. This calculation is best done using the actuarial year-by-year technique. ⁹¹ One may, however, obtain a rough approximation as follows: Assume that the husband would have died at the expiry of his expectation of life and discount the expected inheritance sum with interest to present time. A deduction must then be made for the chance that the wife may not have survived to that time together with any other contingencies affecting inheritance prospects. ⁹² Thus a white male

⁸¹I will refer to husband and wife for sake of convenience. This is intended to include other joint life relationships (father and son; mother and son, etc) where the mortality of the dependant is not negligible. Typically a mongoloid child will be dependent for life but with little prospect for survival beyond age 35.

⁸²SALT79/81 white males.

⁸³SALT79/81 white females.

⁸⁴At 88.

⁸⁵After that age the chance of joint survivorship is nil *ex hypothesis*.

⁸⁶See Koch 'Damages' 281 288 for worked examples.

⁸⁷One does find instances where it is incorrectly argued that the wife's mortality should be ignored because women have longer life expectations than men (see, for instance, Davel `Broodwinner' 363-4; Davel `Skadevergoeding' 84 123).

⁸⁸A good estimate of the joint expectation of working life may be obtained by calculating for each spouse separately the ratio of the expectation of working life to the period to retirement age for the breadwinner. If these ratios are designated Rm and Rf then the ratio for their joint survivorship is given by RmxRf.

⁸⁹For a discussion of the extent to which compensation may be claimed for loss of inheritance prospects see 330.

⁹⁰For a definition of a 'gross multiplier' see 97.

⁹¹See Koch `Damages' 290; see table 21 at 335 for a worked example.

⁹²Davel `Skadevergoeding' 84 123 suggests that if the survivor is older than the deceased and may be expected to die earlier then inheritance prospects should be ignored. Davel

presently aged 40 has an expectation of life of 31 years, ⁹³ an expected age at death of 71 years. The chance that his wife aged 40 will survive until then is 71%. ⁹⁴ An expected inheritance of R100000 implies a present value of R46511⁹⁵ before allowance for the wife's mortality and R46511x0,71=R33023 after allowance for her mortality. In practice there would be a further adjustment for general contingencies to allow for the risks of life that affect the accumulation and preservation of capital. ⁹⁶

[5.5.2] Actuarial year-by-year method: Apart from being a powerful analytical tool this method provides a more precise value based upon the chance in each year that the husband will die and the chance that his wife will still be alive. For instance during the 12 months following the 60th birthday the chance of death for a white male now aged 40 would be 0,020, ie 2%. The chance that his wife would then still be alive is 89%. The chance that she will inherit in that year is 0,020x0,89=0,0178, ie 1,78%. The present value of inheritance of R100000 in that year is R1780⁹⁹ less a discount for interest for 20½ years, 100 giving R1780x0,603=R1073. Using an electronic computer such values are readily assessed for each and every possible 12-month period between ages 40 and 99. The sum of the values gives the value of the *spes* before any deduction for general contingencies. I have calculated this to be R35116 which is 6% higher than the value of R33023 calculated above using gross-multiplier reasoning.

[5.5.3] Retirement lump sums: The gross multiplier approach to inheritance prospects does not yield very good results when the breadwinner will only acquire possession of the asset in years to come. Perhaps the most common example of this is the accrual of a substantial retirement lump sum to civil servants at age 65. Consider the prospect of a lump sum retirement gratuity of R100000 for a coloured male presently age 40. This will only be available for inheritance after age 65. The gross multiplier approach would place on this the same value of R33023 as if the breadwinner had taken immediate possession. The year-by-year approach indicates a nil value for the spes in each year prior to age 65 and gives a total value for the spes on death after age 65 of R15450. Such deferred inheritance prospects are also to be found under trust funds and with aged parents. The year-by-year calculation can be readily extended to cover complicated death and survival contingencies involving 3 and more lives

here fails to appreciate that the expiry of the expectation of life merely marks the point where survival ratios drop below 50%. She treats such chances as nil (see the pseudo ratios in table 5 column A at 88).

⁹³SALT79/81 white males.

⁹⁴The 71 years and the 71% similarity is entirely fortuitous.

⁹⁵Discounting at a net capitalization rate of 2,5% per year compound.

⁹⁶See footnote 455 at 336.

⁹⁷SALT79/81 white males: (73231-71386)/91723.

⁹⁸SALT79/81 white females: (85445+84361)/2/95760 (allows for her survival to mid-year).

⁹⁹R100000x1,78/100.

¹⁰⁰At 2,5% per year compound.

¹⁰¹See footnote 91.

and a variety of different sequences for the occurrence of the deaths. 102

[5.5.4] Awards of damages: In the next chapter I will discuss the concept of consuming interest and capital over the expectation of life of an injured claimant. This concept takes as its premise that the claimant will as a matter of certainty live until the expiry of his expectation of life at which point in time the entire damages award will have been consumed. In terms of this model the heirs have a nil prospect of inheritance because the chance of death prior to expiry of the expectation of life has been assumed to be nil. However, if one considers the year-by-year use of the value of the chance of death one finds a substantial value for the prospect of inheritance. This is so because the chance of early death prior to expiry of the expectation of life is in fact close to 50%. It will take many years to consume the capital but in each of those years there is a very real chance of early death. If one assumes that the award of R100000 will be consumed in accordance with model B under table 6¹⁰³ then for a coloured male aged 40 the value of the chance that his wife, also aged 40, will inherit before he attains age 60 is R20251, ¹⁰⁴ that is to say 13% of the original capital awarded.

It is clear that because of the risk of early death the heirs of the claimant acquire the expectation of a substantial inheritance regardless of what amount the court chooses to award.

Another important conclusion is that the possession of substantial capital provides financial security for the family of a breadwinner claimant in the event of his early death. For this reason he no longer needs the life insurance and widow's pension benefits that may otherwise have been provided by his employer. The value of such death benefits should thus, in many instances, be omitted from any compensation awarded.¹⁰⁵

[5.6] LAW OF LARGE NUMBERS

[5.6.1] Frequency predictions: When an actuary does his reserving for a life office or pension fund he establishes contingency funds¹⁰⁶ on the basis of life-table averages. This is appropriate in the circumstances because life offices and pension funds have numerous members. The statistical law of large numbers¹⁰⁷ tells us that under such conditions averages provide good predictors of what will happen in the future. For this reason the actuary can predict with considerable accuracy for a life office the number of deaths which will occur during the coming years. The law of large

¹⁰²There was time when life offices would buy the contingent rights of beneficiaries to income or capital from trust funds. The valuation considerations formed a part of the syllabus for trainee actuaries (Benz & Tappenden `Reversions & Life Interests'; see too Hooker & Longley-Cook `Life Contingencies' vol 1 87-109; Neill `Life Contingencies' 249-80).

¹⁰³See 100.

¹⁰⁴SALT79/81 coloured mortality.

¹⁰⁵See 60.

¹⁰⁶See 106.

¹⁰⁷The 'central limit theorem' (Levin 'Statistics for Management' 2ed 262-3).

numbers provides not only accuracy but also a measure of the likely size of errors. 108

[5.6.2] Consuming interest and capital: The actuary manages the life office and pension fund financial reserves by thinking in terms of groups of claims. His statistical tables tell him that out of 1000 policyholders 5%, ie 50, will claim accident benefits of R1000 each in 10 years' time. The total liability of the life office in that year will be 50xR1000, ie R50000. The actuary can invest fairly accurately to meet this liability. A single individual equipped with 5% of R1000, the value of the chance, has no hope of benefiting unless he takes advantage of the law of large numbers and uses his R50, 5% of R1000, to buy suitable insurance. He loses the use of his premium but gains the advantage of knowing that his expense will be met if it arises.

[5.6.3] Cross subsidies: When working with large numbers the notion of consuming interest and capital is a valid financial model because those who die early provide the funds for those who live long. The effect of this cross subsidy is that the reserve for a portfolio of life annuitants does not run down over the original expectations of life but is continually extended to take account of the 'end-of-a-rainbow' phenomenon, the Bayesian reassessment of the risk. 111

[5.6.4] Individuals and utility: Statistical prediction tells us what proportion of a group will suffer loss but it cannot tell which members of the group will suffer. The power of an average as a predictor is at its absolute worst when applied to a single individual. In fact it is no predictor at all and it would be an abuse of the information to use it in the sense of a predictor. What the average does indicate to us is the present marginal utility of an uncertain future event. It is the decision criterion, the point at which in the mind of a reasonable person the prospect of gain balances the prospect of loss. 114

[5.6.5] Classical statisticians: Actuarial explanations for lump-sum once-and-for-all awards to individuals are generally unsatisfactory. The wording of many actuarial reports describing damages calculations contemplates a one-man pension fund but fails to explain the relevance of averages and the funding of longevity when there is no cross subsidy with those who die early. Typical of such inappropriate

¹⁰⁸By way of the standard deviation and other such measures.

¹⁰⁹In practice insurance premiums also include allowance for administration expenses and discounts for interest.

¹¹⁰Kemp 'Damages' 3ed 103 'by recourse to both interest and... capital, then the total investment would be exhausted on the death of the last survivor'.

¹¹¹Bayesian reassessments are familiar to jurists in the sense of more accurately assessing the loss in the light of events supervening between date of delict and date of trial (eg *Wigham v British Traders Insurance* 1963 3 SA 151 (W) 156C).

¹¹²Van Rensburg Huldigingsbundel Daniël Pont 384 390-1.

¹¹³Crocker 1980 *TIAA* 517 572 'Knowing the misuse to which life expectancies are prone to be put by lawyers...'.

¹¹⁴See 5 et seq for further discussion of `utility'.

¹¹⁵See hypothetical discussion between judge and actuary quoted at 86.

explanations is the large-numbers model used by Prevett:¹¹⁶

If there had been a very large number of similar individuals of the same age all receiving the same amount, then overall they would have equated to the stated payments, allowing for the operation in due time of compound interest and mortality... if this very large number of individuals made a pool investment of the total of the identical amounts awarded... and if each received from the pool for the remainder of his lifetime the annual loss for which he had been compensated by recourse to both interest and (to the extent necessary) capital, then the total investment would be exhausted on the death of the last survivor'.

The picture we have here is that actuaries apply familiar procedures (ie law of large numbers with cross subsidy) in an unfamiliar context (ie no large numbers, no pooling of experience). These actuarial explanations brand the actuaries as classical statisticians in the sense that chance ('probability' in the statistician's sense) is interpreted as a frequency ratio. In relation to a single individual claimant we need, however, to fall back on very much more intuitive notions of chance.

[5.7] CONCLUSIONS

The standard actuarial calculation proceeds on the basis of the year-by-year application of value of a chance, each year being separately discounted for the risk of early death. The lump-sum present value is the sum of the separate values of a chance and is itself no more than the value of the overall chance. Restitution in the sense of reproducing the lost income is only possible when the risks are very small, as may arise with the dependency of a child. It is otherwise unsound to speak of reproducing the lost income by consuming interest and capital. The use of life annuities for this purpose is generally resisted due to the absence of suitable contracts. 117

A focus on the value of the separate chances of life and death permits solutions to complex problems involving retirement benefits and inheritances. Actuaries are specially trained to handle the complex arithmetic of life-table chances.

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¹¹⁶Kemp 'Damages' 3ed 103. I have rearranged some of the phrases for ease of reading.

¹¹⁷See 118